Coalitional bargaining with an informed player*

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Abstract

In this paper, we consider a model of coalition formation in which one player has private information about her outside option. This player is also essential in that no coalition not including her can obtain any value. Values of coalitions depend on membership but not on the outside option, which only becomes relevant if someone leaves the bargaining. We show that, in any stationary equilibrium for high enough δ, the informed player never makes an informative or acceptable counter offer. If she rejects an offer from an uninformed player, the game ends. An uninformed player therefore calculates what offer maximizes his expected payoff given the amount he has to give other uninformed members of the coalition.

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1 Introduction

In this paper, we consider a model of coalition formation in which one player (say, Player 1) has private information about her outside option, that is, if $v(.)$ denotes the characteristic function of the game, then $v(\{1\})$ is privately known to Player 1. This differs from settings, such as many bilateral bargaining games, where the private information affects the total surplus available to a non-trivial coalition. It also implies that if Player

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1 foresees that future play might involve her taking her outside option, she should do so immediately.

We assume the Player 1 is essential in the sense that value of any coalition S that does not include her is 0. A player is not allowed to be a member of two or more non-identical coalitions, as is usual in this literature. Thus once Player 1 joins a coalition or takes her outside option, the game ends. This *one-coalition property* was a key feature in [33] and [13]. This implies that there is no incentive for any subset of players to wait for some other subset to leave before they form an alliance, a factor that causes possibly delayed agreement in extensive forms where the rejector of a proposer gets the initiative to make a new proposal (as in [8]). *Formally, we do not have the one coalition property in that if a coalition not including Player* 1 *and at least one other player forms and leaves, the game continues.*

As an example of the setting we have in mind, consider a technology entrepreneur with an idea who is considering setting up her own firm. There are interesting questions of how much she has to disclose of her project in order for other players to write down a characteristic function but we do not address these here. The characteristic function values are considered common knowledge except for Player 1's outside option, which has a commonly known probability distribution. Suppose Player 1 is negotiating with other players with different skills in order to set up this firm. Before she enters the negotiation, she has received an offer for her technology from an established firm. This buyout offer constitutes her outside option for the coalitional bargaining. The one-coalition property is natural in this setting, as is the fact that Player 1 is essential, since without Player 1's idea no other player can get a positive payoff.

We model the negotiation through an extensive form used in [8] where a player makes a proposal consisting of a division of the coalitional worth among the members of the named coalition. The members of the coalition say yes or no in sequence. Player 1 is fixed to be the last person to respond. A player can reject the proposal in which case he gets to make a counter-offer or decide to quit the game. If a player quits, the proposal power goes to the next person in the sequence. Since Player 1 has to be a member of every coalition, she can make the offer if everyone else quits. If she quits too, the game is assumed to end with each player getting his individual worth (we shall assume everyone's individual worth is 0, except for Player 1).

For most of the paper, we assume that Player 1 is able to take her outside option only as a responder, not as a proposer. Since she has to be a member of every proposed coalition for that coalition to have value, this is not a significant restriction. However, for example, [8] allows a player to take his outside option only when proposing, so that one-person coalitions are treated on par with larger ones. [35] discusses these different assumptions and their effect on equilibrium play in a model of complete information bilateral bargaining. He asserts that these are best considered as representations of different institutional frameworks for bargaining.

Our results are derived mainly for sufficiently high values of δ. The equilibrium concept we use is stationary Perfect Bayes' equilibrium. Off-path beliefs do play a role but not a significant one. The definition of Perfect Bayes' equilibrium is adopted from [18]; stationarity is used in many multiplayer bargaining papers and is usually justified on the basis of tractability and simplicity (a formal argument for simplicity of strategies for multiperson unanimity games is made by [9]).

Our main results are described informally in what follows. Perfect Bayes' equilibrium involves some assumption about off-path beliefs. These beliefs do not come into play in our most striking result, namely that (for high δ) any proposal by the informed player must be non-informative. We show that the other kind of equilibrium, where such a proposal is partially or fully informative, is not possible. Given the off-path belief, the equilibrium if an uninformed player makes an offer looks like this: either all responders named accept and the game ends or the informed player rejects and takes her outside option, thus also ending the game. If the informed player makes the offer, this denouement is postponed by one period. Thus the game ends within two periods. The examples in Sections 3 will make clear why this happens and why the informed player does not reveal any information thereby ruling out unravelling of beliefs. Of course, off the equilibrium path, the game could continue for a longer duration. The uninformed player has to trade off his payoff against not only those of the other uninformed players but also against the likelihood of getting nothing if the informed player rejects. This suggests the informed player, when she makes a rejected, uninformative proposal, will seek to transfer the proposal power to an uninformed player who will make the proposal with the highest probability of acceptance by the informed player (keeping other players' responses the same).

In terms of the usual questions asked about bargaining outcomes, our model displays a high degree of inefficiency and not in terms of delay. The inefficiency arises from the uninformed proposer underestimating the outside option of the informed player and thus precipitating the end of the game. This will not happen if the optimal proposal happens to be one where the informed party accepts with probability 1. If an efficient solution is reached, it is possible that the informed player gets more than she would have if her $v(\{i\})$ were commonly known. Thus, a novel contribution of our paper is to show a new source of inefficiency in coalitional bargaining with incomplete information where uninformed players make unacceptable proposals owing to incomplete information, and thereafter the informed player quits the game taking the outside option.[1]

Related literature. Some papers in extensive form models of characteristic function games have been mentioned already. Among the ones left out is [21], which is very

[1] We thank an anonymous referee for pointing this out.

different in approach from this paper. Assuming strict superadditivity, [28] studies a complete information bargaining game with proposers randomly chosen in each round. [31] and [25] study games without discounting and identify equilibria that are in the core of the game, also under complete information.[2]

There are fewer papers on coalition formation with some private information. [16] take an ex ante mechanism design approach and define an ex ante core but do not deal with explicit bargaining protocols. [14] define a different notion of core based on some consideration of blocking and information. There is no bargaining in these papers.

[34] and [29] are the two papers we know of that involve non-cooperative modelling. However, these two papers are very different from ours and neither involves discounting. [34] extend the notion of core to an incomplete information exchange economy by formalizing the coalitional decision to object (to a status quo allocation) via an intra-coalition simultaneous move single period Bayesian game. [29] generalizes this idea of coalitional objection by allowing sequential one-stage intra-coalition unanimity voting, which allows for information transmission among members. [29] further considers an alternating offer intra-coalition repeated bargaining game similar to ours, and formalizes objections that constitute stationary sequential equilibrium under the assumption that: proposals in this bargaining game are never informative.[3] In our paper, we find this property to be a necessary feature of any equilibrium.

Another interesting recent paper that deals with somewhat different issues than ours is [24]. Among the differences from our paper, are their assumptions of verifiable types and a mechanism designer who can punish players, which do not fit our framework. Also [2] study exclusionary commitments (the seller commits to negotiate with a strict subset of buyers) in a complete information game in which there is a sequence of bilateral bargains. Though interesting, it is not related to our current paper.

The literature on bilateral bargaining might provide some more analogues to our work and we provide a selective and brief summary of this literature, omitting complete information papers. The most famous strand of this literature is that related to the Coase conjecture in which the seller (who is uninformed) makes offers and the buyer (who is privately informed) accepts or rejects. If the lowest buyer value exceeds the seller value, a unique, weakly stationary equilibrium exists ([17]). As offers become more frequent ($\delta \to 1$), the seller's offer converges to the lowest buyer value and the game ends almost immediately ([23])- this being known as the Coase conjecture. This is not particularly relevant to our results here since the informed party never makes offers. [22] show that

[2] There is also a large literature on complete information coalitional bargaining in settings with externalities across coalitions. For details see [32].

[3] [29] cites this exogenous restriction of non-informative offers as an application of the "*principle of inscrutability*" proposed by [26]. He also imposes another exogenous restriction, which presumes that players respond using a type dependent cutoff rule. This property, too, is obtained as a necessary property for any equilibrium in our paper.

a similar result can hold if the informed party does make offers, under the condition that such offers, if not accepted, are uninformative. This condition is one that we prove as a characteristic of all stationary PBE in our model. Many variants seek to examine this conjecture (see [15] and [12] in different models). Of more relevance are the models with one-sided incomplete information where the informed party does make offers. [20] construct an equilibrium where the informed player's offers can be informative. Our result is similar, though the setting is very different, to Ausubel and Deneckere's "right to remain silent" ([4]) where the informed player remains silent rather than give away any information. [3] construct a model where the duration of the silence (the other player cannot interrupt) conveys information.

In two-sided incomplete information models such as [10] and [11], with each informed player being of two types, a player keeps making a non-informative offer as part of a randomised strategy until one of them reveals his type and the game becomes a one-sided incomplete information game.[4] The logic of the Coase conjecture then takes over, so that the player who is first to reveal loses all gains from trade as $\delta \to 1$ ([27] shows this last part in his textbook). Similar results are also presented in [1] and other papers built on their reputation model. Notice that in this literature, the war of attrition means the game continues, whilst in our model in the paper, the informed player quits because any attempt to use the information to do better fails in equilibrium.

Finally, as mentioned in the introduction, [8] was one of the early papers to study a non-cooperative model of coalition formation, and we have adopted their protocol where the first rejector among sequential responders becomes the next proposer. However, our model is very different. We have private information for Player 1. Player 1 is also an essential player, which means no coalition without Player 1 being a member can obtain a positive payoff and that once a coalition forms with Player 1 in it (or Player 1 quits) the game is over. The problems arising with subsequent coalitions in [8] do not occur here. When the private information becomes too small to matter, the [8] grand coalition solution becomes a limiting case of ours. In future, whenever we refer to "the complete information bargaining equilibrium", we will mean [8].

2 Model

Consider an economic interaction involving players in $N = \{1, \ldots, n\}$ described by an essential game: (i) $v(N) = 1$, (ii) $v(S) = 0$ if $1 \notin S$, and (iii) $0 \leq v(S) < v(N)$ for all $1 \in S \subset N, |S| \geq 2$. However, the outside option of the essential Player 1, that is, $v(\{1\}) := \pi$ is private knowledge, and is, publicly known to be distributed over $[\eta, 1], \eta \geq 0$

[4][30] focuses on a two player alternating offers bargaining game with discounting where both players have private verifiable types, and bargain over contracts. But he does not focus on general coalition formation like our analysis.

with a cumulative distribution function $F(.)$ that has positive density all over the support.

The players bargain over forming a coalition using a bargaining game with '*sequential offers - rejector proposes protocol*'. So, at each information set, an *active player* (that is, a player who is yet to accept a proposal or quit in the game): either makes a proposal, or else responds to a proposal by either accepting it or rejecting it or else quitting the game altogether. A proposal by any active Player i is a tuple $P := (S, y)$ where: (i) $i \in S$ with S being a subset of the set of all active players containing at least two members, (ii) $y \in \mathbb{R}^{|S|}$, and (iii) $\sum_{j \in S} y_j \leq v(S)$.[5] Whenever such a proposal (S, y) is made, members of S respond to this proposal sequentially according to any exogenous linear order $\succ_r$ defined on N. A proposal is deemed to be accepted if all members of S unanimously accept it. If a proposal (S, y) is accepted, then the highest ranked active player in $N \setminus S$ according to any exogenous linear order $\succ_p$ defined on N, makes the next proposal. If (and only if) a proposal is rejected, all players who have not yet quit the game, incur a period of delay, after which the rejector proposes.[6] The utility loss due to delay of one period is captured using a discount factor $\delta \in (0, 1)$. Any player who chooses to quit while responding to a proposal realizes her outside option without any delay, while the highest ranked active player according to $\succ_p$ makes the next proposal.[7]

A strategy of any Player i in our game, is a list of actions such that for each information set that may arise in the game where i would have to move, it prescribes a feasible action to be undertaken. We use the notion of Perfect Bayes' Equilibrium (PBE) to identify an equilibrium in our bargaining game. A PBE assigns to each information set in the game, say I, an action as well as a belief. This assigned action is the one that the player with move at I, say k, is prescribed to undertake. The set of assigned beliefs are probability distributions on I, which in our particular game of observable actions and perfect recall, translate into probability distributions over the Borel measurable subsets of $[\eta, 1]$. These beliefs must be consistent with Bayes' rule, wherever possible. The action assigned at I must be optimal for k, given her assigned beliefs at I. Finally, in line with [18], we

[5] Note that we allow proposals to offer negative amounts to members. Such proposals would never be accepted in equilibrium, but may facilitate information transmission on equilibrium path. We thank an anonymous referee for suggesting this.

[6] It is assumed to be common knowledge that; if a proposal is accepted, then it becomes a binding contract that is enforceable by courts, which in turn, implies that the proposer must make good on the promised payoff distribution. This assumption ensures that any proposal accepted on the equilibrium path must have a payoff distribution that sums up to the worth of the associated coalition.

[7] In the simplest setting with two players: one informed and one uninformed party - our bargaining game extensive form boils down to the standard alternating offers bargaining game where each player has an option to quit only while responding to a proposal. If a player rejects a proposal, she proposes next after a period of delay. The game ends if either of the players quit or accept. The uninformed parties realize an outside option of zero when they quit. On the other hand, if the informed party quits, she receives an outside option π which is privately known to her. All uninformed parties have a common prior belief about this outside option distributed with a differentiable increasing distribution function $F(.)$ over $[\eta, 1]$.

assume that at all information sets, all uninformed parties hold the same belief about the informed party.[8]

In this paper, we specifically investigate belief stationary PBE in pure strategies. To define these strategies formally, let for any time period t, h^t be the list of actions, proposals and responses that have happened in the game at all periods up till t (excluding period t). Further, for any $j \in N$ and any period t in the game, we define the collection of histories h^t after which j has the move (at period t) as j's information set I_j^t, and let $\mathcal{I}_j^t$ be the set of all possible I_j^t. Also, let A_j^t be the set of all the actions that j can take at period t and finally $\beta_j(I_j^t)$ be a probability distribution over the histories in I_j^t. A pure strategy for any player j is a function $\sigma_j : I_j^t \mapsto A_j^t$ for any t. Further, the set $B_j := \{\{\beta_j(I_j^t)\}_{I_j^t \in \mathcal{I}_j^t}\}_{t\in\mathbb{N}}$ is a belief system of player j. Now, a profile of strategies $\sigma = \{\sigma_j\}_{j\in N}$, and a system of beliefs $B = \{B_j\}_{j\in N}$ constitute a Perfect Bayesian Equilibrium if and only if:

1. For any player $j \neq 1$, any period $t \geq 1$, and any information set $I_j^t \in \mathcal{I}_j^t$, σ_j prescribes a best response, $\sigma_j(I_j^t)$, to $\sigma_{-j} := (\sigma_1, \sigma_2, \ldots, \sigma_{j-1}, \sigma_{j+1}, \ldots, \sigma_n)$ given the beliefs $\beta_j(I_j^t)$.

2. For any player $j \neq 1$, any periods $t, \hat{t} \geq 1$, and any information sets $I_j^t \in \mathcal{I}_j^t, I_j^{\hat{t}} \in \mathcal{I}_j^{\hat{t}}$,

$$\beta_j(I_j^t) = \beta_j(I_j^{\hat{t}}) \implies \sigma_j(I_j^t) = \sigma_j(I_j^{\hat{t}}).$$

3. For any $t \geq 1$, any information set $I_1^t \in \mathcal{I}_1^t$, σ_1 is Markov stationary and depends only on the state consisting of the current proposal, say P, the set of active players remaining in the game, and the beliefs B.

4. Bayes' Rule is used to update beliefs whenever possible.

Our notion of belief stationary PBE is same as the one defined in [17] (see footnote).[9] So, at any two information sets I and I' such that the same uninformed player, say i, has the move at both information sets: if a PBE assigns identical beliefs to i at both information sets, then the assigned action at both information sets must be identical too. Henceforth, in the paper, we refer to a belief stationary PBE as an '*equilibrium*'.

3 Example

Let us denote the informed player with type $\pi \in [\eta, 1]$ as 1_π. We begin this section by

[8] Our notion of PBE is same as the one used in [22].

[9] [17] state that *"Let $\beta(p_t, H_{t-1})$ be the least (inf) value of any buyer to buy in period t. An equilibrium is called "weak-Markov" if $\beta(p_t, H_{t-1})$ depends only on p_t (which implies that $V_S(b, H_{t-1})$ depends only on b). Let $\sigma(b^e, H_{t-l})$ be the seller's probability distribution over prices in period t. An equilibrium will be called "strong-Markov" if it is weak-Markov and in addition σ depends only on b^e. In a strong-Markov equilibrium, players' actions depend solely on the "relevant" part of the history, namely, the seller's beliefs and the current offer."* [17] also allow off the equilibrium path beliefs to depend on earlier rejected offers.

noting the fact that for any equilibrium of this game, and any information set I on the equilibrium path, all uninformed parties share the same belief about the outside option of the informed party 1. This follows from the facts that: (i) past actions of 1 are equally observable across all uninformed players, and (ii) the beliefs of uninformed parties must be formed on the equilibrium path in accordance to Bayes' rule.

For any equilibrium, we define $G^i(I, B)$ for all $i \neq 1$, to be the continuation game which starts from the information set I where Player i has the move to propose, and all uninformed players have the same belief B about the distribution of private type π.[10] Similarly, for any equilibrium, define $G^{1_\pi}(I, B)$ as the continuation game starting from information set I where the informed player of type π has the move to propose, and all uninformed players haver the same belief B about the distribution of private type π. For simplicity of notation, we often: (1) suppress the argument I in the notation for a continuation game wherever the relevant information set is clear from the context, and (2) drop the subscript π (when the relevant outside option is clear from the context), and write this continuation game as $G^1(B)$.

We present the following example, which provides an informal and intuitive exposition of the nature of equilibria in our bargaining game.[11] Consider a bargaining setting where: $v(N) = 1$, $v(12) = 0.9$, $v(13) = 0.45$, $v(1) := \pi \sim unif[0.4, 1]$, and $v(S) = 0$ for all other $S \subset N$. Fix $\delta = 0.8$, and suppose that Player 2 is the first proposer. Further, suppose that:

- Players 2 and 3 always propose formation of coalitions $\{1, 2\}$ and $\{1, 2, 3\}$, respectively. The latter proposal is always accepted by Player 2. Player 1 always makes an uninformative proposal that is rejected by either Player 2 or Player 3.

- Players 2 and 3 accept a proposal if and only if each uninformed member of the coalition to be formed, is offered an amount at least as great as δ times the maximum expected payoff that she can obtain by making a proposal herself. Any type 1_π accepts a proposal if it offers her at least π, or else she quits.

We argue below, informally, that when all players are expected to play in a manner consistent with the description above, no player can benefit by deviating unilaterally. To see this, note that proposing $\{1, 2\}$ is better than proposing $\{1, 2, 3\}$ for Player 2. That is because, the expected payoff by proposing $\{1, 2\}$ and offering any amount y_1 to 1 is $(0.9 - y_1)\text{Prob}(\pi \leq y_1) = \frac{(0.9-y_1)(y_1-0.4)}{0.6}$, which is maximized when $y_1 = 0.65$ (the corresponding probability of acceptance is $\frac{5}{12}$). Thus, the (maximum) expected payoff to Player 2 by proposing $\{1, 2\}$ is 0.104. Arguing similarly, the expected payoff to 3 by proposing $\{1, 2, 3\}$ in a manner that is acceptable to Player 2 is, $\max_x (1 - x)\text{Prob}(\pi \leq$

[10] It can easily be seen that B denotes a pair consisting of a measurable subset of $[\eta, 1]$, and a distribution over it.

[11] We also provide a two player example in subsection 6.1 of Appendix.

$x) - 0.8 \times 0.104 = 0.0666$, with the maximizing x value being 0.7, and the corresponding probability of acceptance being 0.5. Now consider the maximum expected payoff that Player 2 can get by deviating and proposing $\{1,2,3\}$ in an acceptable manner. This value is given by $\max_x (1-x)\text{Prob}(\pi \leq x) - 0.8 \times 0.0666 = 0.0966$ which is less than 0.104, implying that proposing $\{1,2\}$ is more profitable for Player 2 than proposing $\{1,2,3\}$. Finally, Player 3 will not find it profitable to deviate and propose $\{1,3\}$ because the maximum available surplus to negotiate over is $v(\{1,3\}) - 0.4 = 0.05$, which is less than 0.0666.

Finally, consider the informed party 1. Note that Player 1's proposal can never be informative on the equilibrium path, because an informative proposal would divide the support into at least two subintervals. For example, suppose type $\pi \in [0.4, 0.7]$ is prescribed to make an acceptable proposal to Player 3 offering her 0.7 while type $\pi \in [0.7, 1]$ is prescribed to make an unacceptable proposal which will be rejected by either of Players 2 and 3. Then, the type $\pi \in [0.7, 1]$ would be offered 0.85 in the next period, implying that all types in $[0.4, 0.7]$ would deviate to mimic the types in $[0.7, 1]$.

Now, as we show later in Lemma 1, any non-informative offer to be made by the informed party must be an unacceptable offer. Note that such an offer must be addressed to Player 3 as she is the uninformed player prescribed to propose the maximum amount 0.7 to Player 1 in the next period. Thus, if player 1 is the first proposer, she makes an unacceptable proposal, and so, her equilibrium payoff is: 0.56 if her type lies in $[0.4, 0.7]$, or else $\delta\pi = 0.8\pi$ (obtained by quitting in the second period).

As we argue formally in the forthcoming results, these kinds of proposal and acceptance behaviours are the only kinds permissible in any equilibrium. Also note that the equilibrium expected payoffs, when Player 2 is the first proposer, are $(0.65, 0.104, 0)$; and the coalition $\{1,2\}$ forms on the equilibrium path immediately with probability $\frac{5}{12}$, or else the informed party quits. Compare this to a complete information setting where π is known to be, say, 0.7, and all other information remains unchanged. As $\delta \to 1$, we can obtain the efficient limiting equilibrium payoffs when Player 3 is the first proposer (from [8]) to be $(0.7, 0.2, 0.1)$. Notice that uninformed Players 2 and 3 do badly under incomplete information, and the equilibrium outcome is inefficient with positive probability irrespective of who proposes first.

Finally, note that the exact choice of an equilibrium unacceptable proposal by the informed party in our three player example, must be made optimally so that the proposer power gets passed on to the uninformed party, who offers the largest amount to the informed party in the next period. This feature does not arise in any two player example (one such example is provided in the Appendix), as the choice to address an unacceptable proposal becomes trivial in this case.

4 Results

We now present the formal results underlying the example above. First, we prove the following claim which states that on the equilibrium path of our bargaining game, there cannot be two types of informed party who make different proposals but get the same equilibrium payoff.[12]

Claim 1. *On the equilibrium path, types of informed party making non-identical proposals, get non-identical equilibrium payoffs.*

Proof of Claim 1: Fix any equilibrium σ, and any δ sufficiently close to 1. Suppose there exists an information set I on the equilibrium path such that two types of informed party $1_{\pi_l}, 1_{\pi_h}, \pi_l < \pi_h$ are prescribed by σ to make different proposals, say L and H, respectively; but get the same equilibrium payoff X in the continuation game starting from I. We consider the following three cases.

Case 1: *L and H are unacceptable proposals.*
In this case, both proposals, L and H, are rejected on the equilibrium path. If any of these types, 1_{π_l} and 1_{π_h}, realizes her expected payoff by *making* an acceptable proposal in future, then she could have realized a greater expected payoff from proposing the same coalition and the same proposal at the information set I. This would contradict our supposition that L and H are equilibrium prescriptions.
If both types, 1_{π_h} and 1_{π_l}, realize their equilibrium payoff by *accepting* a proposal in future, then some uninformed party offers them the same amount in spite identifying them as different types on the equilibrium path. This is possible only when the the private information is trivial (that is, the maximum possible outside option is sufficiently small), and the uninformed parties view our bargaining game essentially as the game of complete information bargaining. But, in that case, making unacceptable proposals L and H at information set I cannot be equilibrium actions, which is a contradiction.
Thus, the only remaining possibility under this case is that both 1_{π_l} and 1_{π_h} realize their equilibrium payoffs by quitting on the equilibrium path. But this contradicts our supposition that both types get the same equilibrium payoff X.

Case 2: *Only one of the two proposals, L and H, is unacceptable.*
In this case, any one of the types makes an unacceptable proposal, while the other makes an acceptable proposal to get the same equilibrium payoff X. Note that L cannot be unacceptable since then, by the logic mentioned above in Case 1, she must get her equilibrium payoff by quitting on equilibrium path, and so, $X = \delta\pi_l < \delta\pi_h$. But this implies that X cannot be the equilibrium payoff of Player 1_{π_h} since she can guarantee herself $\delta\pi_h$ by making an unacceptable proposal. Therefore, this possibility (that is, Case

[12] We thank one anonymous referee for raising this issue.

2) can hold true only if L is an acceptable proposal but H is an unacceptable proposal. Hence, by the logic mentioned in the previous Case 1, 1_{π_h} must realize her equilibrium payoff by quitting on the equilibrium path, and so, $X = \delta\pi_h$. Further, by the mimicking restrictions embodied in the equilibrium notion, it must be that equilibrium payoff of all types $1_{\pi'}$ with $\pi' \in [\eta, \frac{X}{\delta})$ must be equal to X. And so, since all such types have an outside option $\pi' < \pi_h$, by the previous arguments, they must make an acceptable proposal.

But this implies that upon observing an unacceptable proposal on the equilibrium path, the uninformed parties realize, that is, update their beliefs to $\pi \in [\pi_h, 1]$, which means that their equilibrium proposal after rejecting H must offer informed party an amount greater than π_h (or else her counteroffer has zero probability of acceptance), which would be accepted by 1_{π_h}. This contradicts our earlier inference (obtained in Case 1) that any type making an unacceptable proposal must quit on the equilibrium path.

Case 3 *Both L and H are acceptable proposals.*

In this case both acceptable proposals must propose formation of different coalitions, say S^l and S^h respectively, but demand the same amount X. Since both these proposals are accepted on the equilibrium path in spite of unravelling information about types, it must be that private information about types is too low to matter. That is, if any uninformed party were to reject any of these offers and make an equilibrium counteroffer as in a complete information bargaining game (that is, propose a largest average worth maximizing coalition T^*, offering informed party $c := \frac{\delta v(T^*)}{1+(|T^*|-1)\delta}$); then this counteroffer would be accepted with probability 1. This implies that acceptance threshold of each uninformed party is c, and so, $X = \frac{c}{\delta}$. Now, by the arguments above, to eliminate mimicking incentives, it must be that all informed types with outside option in $[\eta, \frac{X}{\delta}]$ must get the payoff X. But then, there exists a measurable set of types (in the open interval $(\frac{c}{\delta}, \frac{c}{\delta^2})$) that would be expected by any uninformed party to reject the aforementioned complete information equilibrium counteroffer (to form T^*), and this, in turn implies that the counteroffer will not be accepted with probability 1. Hence, we get a contradiction to our earlier inference of the private information being too low.

Since we get a contradiction in all possible cases, the result follows. □

Now, we use Claim 1 to prove the lemma below, which establishes that for any equilibrium of the game, there can be no information set where any type of the informed party makes an informative proposal (a proposal that leads to any updating of beliefs held by uninformed parties by Bayes' rule).

Lemma 1. *The informed party never makes an informative proposal on the equilibrium path.*

Proof: Suppose there exists an equilibrium σ such that there exists some type $1_{\tilde{\pi}}$ who is prescribed to make an informative offer (one that leads to updating of beliefs of uninformed parties) at some information set on the equilibrium path. Fix I to be the first such information set on the equilibrium path (that is, at all earlier information sets on the equilibrium path, the informed party made non-informative offers, if called on to make offers), and consider the continuation game $G^1(I, B)$. Note that, by construction, the belief B must have a support $J \subseteq [\eta, 1]$, and so, we can define $a := inf\ J$ and $b := sup\ J$.

Now for each type 1_π with $\pi \in J$, define x_π to be the equilibrium payoff from playing according to σ in the aforementioned continuation game. By construction, $\tilde{\pi} \in J$, and so there must exist a different type $\hat{\tilde{\pi}} \in J$ such that $1_{\hat{\tilde{\pi}}}$ is prescribed by σ to make an offer which is different from the one that $1_{\tilde{\pi}}$ is to make (or else we would contradict our supposition that $1_{\tilde{\pi}}$ makes an informative offer at information set I). Now, at information set I: either both these types $1_{\tilde{\pi}}$ and $1_{\hat{\tilde{\pi}}}$ make different acceptable offers, or they make different unacceptable proposals, or else any one makes an acceptable proposal while the other makes an unacceptable proposal. Therefore, if we define $\bar{J} \subseteq J$ to be the set of types of the informed party in J, who are prescribed to make an unacceptable proposal at I, we get three possibilities: (i) $\bar{J} = \emptyset$, (ii) $\bar{J} = J$, and (iii) $\bar{J} \neq \emptyset, \bar{J} \neq J$.[13] We consider each possibility in the following discussion as a different case.

Case (i). In this case, our supposition is that: (a) all types in J are prescribed to make acceptable proposals, and (b) there exists at least one type $1_{\tilde{\pi}}$ whose acceptable proposal is informative.[14] We argue below that for all types $\pi \in J$, $x_\pi = \kappa$ where κ is a non-negative real constant. If not, then there exists at least a pair of types in J, with different equilibrium payoffs (and hence, different prescribed acceptable proposals) implying that the type with lower equilibrium payoff has a profitable unilateral one-deviation in mimicking the other type's equilibrium action at I, which would contradict our supposition of σ being equilibrium.

Now the acceptable proposal, say P', that is to be made by $1_{\tilde{\pi}}$ at I is informative if and only if there exist disjoint subsets J_l, J_h of J, where each type in J_h makes the acceptable proposal P' (that is, $\tilde{\pi} \in J_h$), and all types in J_l make some other acceptable proposal $Q' \neq P'$. Therefore, by Claim 1, there exist types $\pi'_h \in J_h$ and $\pi'_l \in J_l$ such that $x_{\pi'_h} \neq x_{\pi'_l}$, which leads to a contradiction to the aforementioned conclusion that $x_\pi = \kappa, \forall\, \pi \in J$.

Case (ii). In this case, our supposition is that: (a) all types in J make unacceptable proposals, and (b) there exists at least one type $1_{\tilde{\pi}}$ whose unacceptable proposal is informative. Suppose that $1_{\tilde{\pi}}$ proposes some unacceptable proposal P. Now, in a manner similar to the previous case (i): the unacceptable proposal P prescribed to be made by $1_{\tilde{\pi}}$

[13]Given σ, an unacceptable proposal made by informed Player 1 is a tuple (S, y) such that there exists some $j \in S \setminus \{1\}$ such that σ prescribes j to not accept this proposal.

[14]Given σ, a proposal (S, y) is acceptable if it is not unacceptable.

at I, is informative if and only if there exist disjoint subsets J_l, J_h of J, where each type in J_h makes the unacceptable proposal P (that is, $\tilde{\pi} \in J_h$), and all types in J_l make some other unacceptable proposal $Q \neq P$. Therefore, by Claim 1, there exist types $\pi'_h \in J_h$ and $\pi'_l \in J_l$ such that the equilibrium payoffs $x_{\pi'_h} \neq x_{\pi'_l}$. Hence, as argued earlier, either of these two types has the profitable unilateral one-deviation to mimic the other on the equilibrium path. And so, once again, we get a contradiction to σ being an equilibrium.

Case (iii). In this case, our first supposition is that there exist a pair of types $1_{\pi'}, 1_{\pi''}$ such that: (i) $\pi' < \pi''$, (ii) $\pi' \in \bar{J}$, and (iii) $\pi'' \in J \backslash \bar{J}$. Now, if $x_{\pi''} > x_{\pi'}$, then the type $1_{\pi'}$ has a profitable unilateral one deviation to mimic type $1_{\pi''}$, and make the same acceptable proposal as the one prescribed by σ to $1_{\pi''}$. On the other hand, if $x_{\pi''} < x_{\pi'}$, then it must be that $1_{\pi'}$ does not quit on the equilibrium path (or else $x_{\pi'} = \delta\pi' < \delta\pi'' \leq x_{\pi''}$); and so, it follows that there exists a profitable unilateral deviation by type $1_{\pi''}$ where she mimics the type $1_{\pi'}$ (using the same argument as before). Therefore, we get that $x_{\pi''} = x_{\pi'}$, which contradicts Claim 1.
Thus, we can infer that for all $\pi' \in \bar{J}$ and all $\pi'' \in J \backslash \bar{J}$, $\pi' \geq \pi''$. Now define $d := sup\ J \backslash \bar{J}$, and note that by construction $d \in (a, b)$. Therefore, we get that σ prescribes the informed party to make an acceptable proposal if her outside option $\pi \in L^J := J \cap (-\infty, d)$; and make an unacceptable proposal if her outside option $\pi \in H^J := J \cap (d, \infty)$. Now as argued earlier in Case (i), all types 1_π with $\pi \in L^J$ must get the same equilibrium payoff κ', and (**a**) $\kappa' \geq \delta d$. Now if there exists a $\hat{\pi} \in H^J$ such that $x_{\hat{\pi}} < \kappa'$, then $1_{\hat{\pi}}$ has profitable unilateral one deviation to mimic any type $1_{\pi'}$ with $\pi' \in L^J$ and get the higher payoff κ'. Therefore, σ is an equilibrium only if (**b**) $x_{\hat{\pi}} \geq \kappa', \forall\, \hat{\pi} \in H^J$. Now, σ must ensure that there is no profitable unilateral one deviation available to any type $\pi \in L^J$, where she mimics a higher type $\hat{\pi} \in H^J$. This would be true only if σ prescribes all types in H^J to - not only make an unacceptable proposal at information set I - but also quit the game at the consequent response node on the equilibrium path. That is, (**c**) $\forall \hat{\pi} \in H^J$, $x_{\hat{\pi}} = \delta\hat{\pi}$. Therefore, from (**b**) and (**c**) it follows that for all $\hat{\pi} \in H^J$, $\delta\hat{\pi} \geq \kappa'$; and so (**a**) implies (in limit) that $d = \frac{\kappa'}{\delta}$.
However, this implies that at the information set that arises on the equilibrium path with positive probability after an uninformed party rejects an unacceptable proposal made at information set I, the beliefs of uninformed parties are updated in accordance with Bayes' rule where they now believe that $\pi \in H^J$. Let $e := inf\ H^J$, and note the $d \leq e$. Therefore, from (**c**) it follows that: upon observing an unacceptable proposal on the equilibrium path, at least one uninformed party makes a proposal offering some $\zeta \leq e$ to the informed party, in response to which the informed party quits. However, doing so gives this uninformed party an equilibrium expected payoff 0. Hence, she has a profitable unilateral one deviation of offering some $\zeta' > e$ (upon observing an unacceptable proposal on the equilibrium path), which gives her a positive expected payoff as informed types in

the interval (d, ζ') would accept. Thus, we get a contradiction to σ being an equilibrium. $\square$

Lemma 1 above, implies that for any equilibrium of this game, the informed player must make the same proposal irrespective of her outside option at any information set where she is called upon to make an offer. In other words, no matter what the outside option, the informed party always chooses a passive strategy of bargaining with an uninformed party *only while responding.* We show below that if the discount factor is sufficiently high, then this non-informative proposal must be unacceptable to the uninformed parties.

Lemma 2. *As $\delta \to 1$, there exists no equilibrium in which the informed party makes an acceptable proposal.*

Proof: Suppose not. That is, suppose that there exists an equilibrium σ such that the informed party 1 makes an acceptable proposal and $\delta \geq \eta$. By Lemma 1, such a proposal must be non-informative. Further suppose that: (i) under σ, 1 makes such an acceptable proposal P, for formation of a coalition $1 \in S \subseteq N$, at an information set where the informed party has rejected a proposal by the uninformed party, and (ii) this acceptable proposal gives 1 a payoff of x. Since every informed party 1_π at this information set can guarantee herself $\delta\pi$ payoff by making an unacceptable proposal, we can infer that $x \geq \delta$. Further, by Bayes' rule, upon observing the above mentioned acceptable proposal (instead of a termination of game caused due to informed party quitting); the uninformed parties must believe that $\pi \leq \delta x$ before responding to this proposal. Now consider the one deviation where an informed party rejects this counteroffer, and then makes an equilibrium counteroffer that offers 1 an amount $y_1 \leq \delta x$.[15] Note that if $y_1 < \delta x$, then by our supposition, all types of informed party would reject this counteroffer, leading to a period of delay without any further updating of beliefs. Thus σ would constitute an equilibrium only if $y_1 = \delta x$. Therefore, the uninformed parties would expect this counteroffer to be accepted by Player 1 for sure, and so, by the complete information bargaining equilibrium logic, each uninformed party can get at least $\frac{v(S)-\delta x}{1+(|S|-2)\delta}$. Therefore, the proposal P by 1 would be acceptable only if each uninformed party is offered at least $\frac{\delta[v(S)-\delta x]}{1+(|S|-2)\delta}$. Thus,

$$x \leq v(S) - \frac{\delta(|S|-1)[v(S) - \delta x]}{1 + (|S|-2)\delta},$$

[15]Note that, if $y_1 > \delta x$; then this implies that the information (that $\pi \leq \delta x$) unravelled on the equilibrium path, has no impact on the equilibrium proposal of the uninformed parties at this continuation game (which is why they offer the informed party an amount y_1 that is strictly greater than her maximum possible believed outside option). This can constitute equilibrium play only if the incompleteness of information ceases to matter at this continuation game, and *all* players behave as in the complete information bargaining equilibrium where the outside option of the informed party is publicly known to be 0. But if this is true, then there can never be a rejection of proposal by informed party on the equilibrium path. Hence, we arrive at a contradiction to our supposition.

which implies that $\delta \leq x \leq \frac{v(S)}{1+(|S|-1)\delta}$.[16] Since this inequality must hold for all δ values, in limit as δ goes to 1, we get that $1 \leq \frac{v(S)}{|S|}$ which is a contradiction (as $v(S) \leq 1$).

Finally, consider the only other remaining possibility that 1 is the first proposer in the game. Belief stationarity requires that prescription of σ to 1 at the initial information set, should be same as that at the information set analyzed above. And so, the result follows. □

The following lemma builds upon Lemma 1 and Lemma 2, to establish that the informed party never makes a counteroffer on the equilibrium path.

Lemma 3. *As $\delta \to 1$, the informed party never makes a counteroffer on the equilibrium path.*

Proof: Fix any equilibrium σ, and suppose there exists a type of the informed party $1_{\pi'}$ who rejects an offer and makes a counteroffer on the equilibrium path. Let I be the earliest of such information sets; that is, at all earlier information sets on the equilibrium path, no type of the informed party rejects and makes a counteroffer. Therefore, by Lemma 1, the equilibrium belief prescribed by σ to I must have an interval support, say, must be $[a, b]$.[17] Further, for each type π, let x_π denote the equilibrium expected payoff to the informed party 1_π when all players play σ in the continuation game starting from the information set I. Finally, let y be the sure amount that $1_{\pi'}$ would realise by accepting at I.

Therefore, $x_{\pi'} \geq \max\{y, \pi'\}$, which, by Lemma 2, implies that $1_{\pi'}$ must accept a proposal to end the game on the equilibrium path in this continuation game. This implies that all types of informed parties 1_π with $\pi \leq \min\{b, x_{\pi'}\}$ must get the same equilibrium expected payoff $x_{\pi'}$ in this continuation game, or else they would have a profitable unilateral one-deviation of mimicking $1_{\pi'}$ to get a greater payoff or vice-versa. This, in turn, implies that all Players 1_π with $\pi \leq \min\{b, x_{\pi'}\}$ must, on the equilibrium path, take the same actions as $1_{\pi'}$, or else their types would get revealed implying that not all types would get the same equilibrium payoff .[18]

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[16] The inequality implies that $x\left[1 - \frac{\delta^2(|S|-1)}{1+(|S|-2)\delta}\right] \leq \frac{(1-\delta)v(S)}{1+(|S|-2)\delta} \Leftrightarrow X[(1-\delta) + (|S|-1)\delta(1-\delta)] \leq (1-\delta)v(S)$.

[17] Note that the equilibrium belief at I must have an interval support. That is because: if the equilibrium belief at I does not have an interval support, then it must have a support, say J, which is a collection of disjoint intervals. And so, there must exist types π^*, π^{**} such that $\pi^* \neq \pi^{**}$, $\pi^* \in J$, and $\pi^{**} \notin J$. Therefore, at some previous information set $\hat{I}$ on the equilibrium path, σ must have prescribed 1_{π^*} to reject and make a counteroffer while prescribing $1_{\pi^{**}}$ to either accept or quit. But then I cannot be the earliest information set where some type of the informed party rejects and makes a counteroffer. This contradicts our supposition in the proof. In fact, we can infer from Lemma 1 that this interval support must contain all possible types, that is, be $[\eta, 1]$.

[18] As mentioned earlier in footnote 15, there is a possibility where offer of uninformed parties on the equilibrium path does not depend on the beliefs held by them. This possibility arises when $x_{\pi'}$ is the equilibrium proposer payoff of the complete information version of our bargaining game. In other words,

Note that if there exists any type of the informed party $1_{\hat{\pi}}$ with $\hat{\pi} \in (x_{\pi'}, b]$ who is also prescribed to accept a proposal on the equilibrium path in the continuation game starting from I; then $x_{\hat{\pi}} \geq \hat{\pi} > x_{\pi'}$, which implies that $1_{\pi'}$ has the profitable unilateral one-deviation to mimic $1_{\hat{\pi}}$ and get a higher payoff. Therefore, if σ is a PBE, it must be that all informed parties with types in $\pi \in (x_{\pi'}, b]$ must quit at I.
Thus, on the equilibrium path, if uninformed parties observe that game has not ended after the proposal made at information set I, then they update their beliefs to types being distributed in $[a, \min\{b, x_{\pi'}\}]$ by Bayes' rule. This means that at all the information sets on the equilibrium path subsequent to I, uninformed parties must not offer the informed party a sure amount greater than y, implying that $x_{\pi'} \leq \delta y < y$.[19] Thus, we get a contradiction. □

Lemma 1, Lemma 2, and Lemma 3 leads us to the main result of this paper presented below. It states that, as δ goes to 1, any informed party would accept a proposal on the equilibrium path if and only if she is offered an amount equal to her outside option.

Theorem 1. *As $\delta \to 1$, for any $\pi \in [\eta, 1]$, the Player 1_π accepts a proposal on the equilibrium path if and only if it offers her an amount greater than or equal to her outside option π.*[20]

Proof: By Lemma 3, on the equilibrium path, the informed party either accepts a proposal or else quits. Therefore, at any information set on the equilibrium path, if a type of the informed party is prescribed to quit, then her outside option must not be less than the sure amount she was offered, and if she is prescribed to accept then her outside option must not be greater than the sure amount she was offered. Hence, the result follows. □

in this possibility, the private outside option is believed by the uninformed parties to be too low to matter, implying that $x_{\pi'} = \max_{S \subseteq N} \frac{\delta v(S)}{1+(|S|-1)\delta}$. But in this case, as shown in [8], on the equilibrium path in every possible continuation game, the uninformed parties must offer the informed party an amount equal to $\max_{S \subseteq N} \frac{\delta v(S)}{1+(|S|-1)\delta}$, and so there can never be a rejection of such an offer by the informed party on the equilibrium path. This contradicts our supposition.

[19] This would follow from maximization of expected payoff over a smaller interval with the same lower bound. To see a formal version of this reasoning, consider any well behaved functional $h(.)$, and a continuous probability density function $g(.)$ (with an associated distribution function $G(.)$) such that $M(t) := \int_a^b h(t,x)g(x)dx$ is well defined for all $t > 0$. Let m^* solve the problem $\max_{t \in [a,b]} M(t)$. Further, for any $c \in (a, b)$, let m_c^* solve the problem $\max_{t \in [a,c]} \tilde{M}_c$, where $\tilde{M}_c := \int_a^c h(t,x)g(x|x < c)dx$. Now, suppose that there exists a $\bar{c} \in (a, b)$ such that $m^* < m_{\bar{c}}^*$. Since the conditional probability $g(x|x < \bar{c}) = \frac{g(x)}{G(\bar{c})}$ for all $x \in [a, \bar{c}]$, we can infer that $m_{\bar{c}}^*$ maximizes $\int_a^{\bar{c}} h(t,x)g(x)dx$, while m^* does not. Further, by construction, $m_{\bar{c}}^* \leq \bar{c}$, and so, $m^* \leq \bar{c}$, which implies that all types in $(\bar{c}, b]$ would reject any proposal offering either of m^* and $m_{\bar{c}}^*$. Hence, if $\pi \in (\bar{c}, b]$, then the equilibrium payoffs to an uniformed party from offering m^* Player 1 and offering $m_{\bar{c}}^*$ to Player 1, are equal. Thus, we can infer that $M(m_{\bar{c}}^*) > M(m^*)$, which is a contradiction. Therefore, we get that $m_c^* \leq m^*$ for all $c \in (a, b)$.

[20] Ties may be broken in any arbitrary way. It is inconsequential as our prior belief is a density function.

Lemma 1 presents an emphatic result that there is no information unravelling on the equilibrium path, irrespective of the value of δ, or the order of proposers or responders. Lemma 2, Lemma 3 and Theorem 1, on the other hand, are asymptotic results that hold in limit as δ goes to 1. However, these results too, are robust to any variation in order of proposers or responders. Further, for any value of the discount factor δ, there can exist multiple belief stationary PBE of our bargaining game.[21]

5 Discussion

5.1 A model variation

Our bargaining game allows players to quit while responding, and not while proposing. This is the typical practice in the literature on bilateral bargaining with an outside option. [35] notes that breakdown of negotiations has been modelled by allowing players to quit while responding in [7], [36], [5], and [19]. The celebrated paper [6], too, considers only the possibility of quitting while responding when discussing the impact of outside options on their result.

We discuss below a variant of our game where the players can only quit while proposing. Without loss of generality, we restrict ourselves to the case where the informed Player 1 is the first proposer. When players can quit while proposing, there can be multiple *trivial* equilibria with the same equilibrium outcome where all types of informed players quit at the beginning of the game. We present below a result which shows that there can be only one *non-trivial* equilibrium outcome. As can be seen below, in this equilibrium outcome, some types quit while other types play out the complete information bargaining equilibrium path.

Proposition 1. *Suppose that: (i) players can quit only while proposing, (ii)* $\eta < c := \max_{T \subseteq N} \frac{v(S)}{1+(|S|-1)\delta}$*, and (iii) the informed party is the first proposer. The unique equilibrium path where the informed party does not quit immediately is as follows.*

- *All types in* $(c, 1]$ *quit at the beginning of the game.*
- *All types in* $[\eta, c]$ *propose the largest coalition* S^* *which solves* $\max_{T \subseteq N} \frac{v(S)}{1+(|S|-1)\delta}$*, and offer all uninformed members of* S^* *the amount* δc*.*
- *This proposal is accepted by the uninformed members of* S^**.*

Proof:

[21] Theorems 4 and 5 of a working paper version of this manuscript (to be found at `https://www.academia.edu/91966967/Coalitional_bargaining_with_private_information`)present two classes of such belief stationary PBE.

Proof of Necessity: Fix any equilibrium σ such that there exists some type of informed party who does not quit at the beginning of the game. Now, if the equilibrium payoffs for all types of informed party are equal to their respective outside options, then all types must quit at the beginning of the game, which would be a contradiction to our supposed equilibrium σ. Therefore, there must exist an 1_π such that her equilibrium payoff $x_\pi > \pi$. Thus, 1_π cannot realize her equilibrium payoff by quitting on equilibrium path. Hence, she must make an acceptable proposal right at the beginning of the game (as the first proposer), because making an unacceptable proposal would merely cause an extra period of delay in realizing her equilibrium payoff. As a result, all types of informed party with outside option $\pi' < \pi$, $x_{\pi'} \geq x_\pi$, or else they can deviate by mimicking the type 1_π. Similarly, if there exists a type $1_{\pi'}$ such that $\pi' < \pi$ and $x_{\pi'} > x_\pi$, then by the same argument as above, $1_{\pi'}$ must be prescribed to make an acceptable proposal, and so, 1_π can mimic $1_{\pi'}$ to deviate on the equilibrium path. Therefore, it must be $x_\pi = x_{\pi'}$ for all $\pi' \in [\eta, \pi]$. Further, by applying the same arguments, we can infer that $x_\pi = x_{\pi'}$ for all $\pi' \in [\pi, x_\pi]$. Thus, there exists a $\theta \in [\eta, 1]$ such that all types in $[\eta, \theta]$ make an acceptable proposal on equilibrium path, which gives them equilibrium payoff of θ, and all types in $(\theta, 1]$ quit as the first proposer. Hence, the game ends in the first period itself.
Now, consider any acceptable proposal P which is made on the equilibrium path with positive probability at the beginning of the game. As argued above, such a proposal must demand for the informed party the amount θ, and must be accepted by uninformed members even after updating their beliefs to a conditional distribution G over $[\eta, \theta]$. Further, the informed party must be prescribed by σ to make this same proposal P (giving her θ) at any other information set, say I, where uninformed parties have the belief that types are distributed with G on $[\eta, \theta]$; and again, this proposal P would be accepted by all uninformed parties in this continuation game starting from I, giving the informed type an equilibrium payoff of θ (in this continuation game).
Therefore, upon receiving any acceptable proposal P at the beginning of the game, any uninformed party j in the coalition S^P associated with P, can reason out that if she makes a counteroffer offering the informed party $\delta\theta$, then such an offer would be accepted by all types of informed party. Further, any such counteroffer offering Player 1 less than $\delta\theta$ would be rejected by all types of informed party. Since making an unacceptable proposal can never be optimal for any uninformed party (as it delays resolution without affecting beliefs about the type of the essential Player 1), j must make an acceptable counteroffer after rejecting P. And so, the decision problem of choice of best possible acceptable proposal in this continuation game (following the deviation of rejection of P) would be identical to that in the complete information bargaining game. Since any $j \in S^P$ can always propose an acceptable proposal to form S^P while offering $\delta\theta$ to 1,

from the necessity results of [8], we can infer that

$$S^P = S^* \text{ and } \theta = \frac{v(S^*)}{1 + (|S^*| - 1)\delta} = c.^{22}$$

Hence, the result follows.

Proof of Sufficiency: To show the sufficiency, we show that the path of play described in the statement constitutes an equilibrium with suitably chosen off the prescribed path beliefs. For the purpose of this proof, we assume that at any off the prescribed path information set, all uninformed types believe that the informed party is of the lowest possible type η. Thus, no informed party in $[\eta, c]$ has any incentive to make any proposal other than the one described or to quit altogether. Similarly, it is easy to see that no informed party in $(c, 1]$ has any incentive to make a proposal instead of quitting immediately. Finally, as argued in the proof of necessity, no uninformed party in S^* can be better off in deviating to reject and making an unacceptable offer. So any deviation to reject must be followed by an acceptable proposal, and using the arguments of [8], we can infer that the best such proposal can only give her the payoff c. Thus, such a deviation can never be profitable, and hence, the result follows. $\square$

A natural question that would arise is: what happens to the equilibrium in this variant of our bargaining game when $\eta \geq \max_{T \subseteq N} \frac{v(S)}{1+(|S|-1)\delta}$? In that case, we can have only one equilibrium outcome where all types quit at the beginning of the game.[23] Therefore, an alteration in the extensive form that allows players to quit while proposing, induces a change in the equilibrium path where the informed party now makes an acceptable proposal with positive probability (unlike informed party never making an acceptable proposal in our original model). Thus, if we change the extensive form (the model) of the game, the results change as described. This is not unexpected.[24]

Finally, one might consider a different variant of our model where players can quit while proposing as well as responding. In such a game the aforementioned equilibrium path in Proposition 1 cannot exist. This is because, this equilibrium requires uninformed parties to offer δc to the informed party when they get the proposer power and believe that the private outside option is in the interval $[\eta, c]$. This means that informed parties in $(\delta c, c)$ would now find it optimal to reject this proposal and realize their higher outside

[22] The exact formal arguments are a slightly modified version of the proof of [8]. The proof is available upon request. Further, there may be different types of informed parties who may propose different S^* coalitions, but they must be of the same size.

[23] Note that such an equilibrium cannot exist in our original game where players can quit only while responding. This is because quitting in our model can only give a payoff of δ times the outside option (instead of the outside option itself when players can quit while proposing).

[24] We thank an anonymous referee for raising the question of alteration of quitting power in our game; and the Editor for suggesting an interpretation to the modification of the results due to this alteration.

option immediately.

5.2 Order of response

Our major results, Lemma 1, Lemma 2 and Lemma 3, illustrate the qualitative nature of our equilibrium results, and are *not* affected by any change in order of responders (or order of proposers) in our protocol.

5.3 Other papers

It is an interesting question whether the equilibrium payoffs obtained in our game belong to the credible core of [14] or the signalling core of [29].[25] We note that even in the complete information case stronger conditions like convexity in [8] and the condition P with the one coalition property in [13] are required to ensure core membership of equilibrium payoffs. We also note that both the credible core and signalling core require proposals to be contingent on types, which does not hold in our framework. Also, due to incomplete information in our setting, any rejection by the informed party would reveal private information, leading to equilibria (as δ goes to 1) where the informed party always quits after rejecting. This reduces the veto power of the informed party, and so, results like that of [37] do not hold.

6 Appendix

6.1 A Two Player Example

Consider a two player bargaining setting where: $v(N) = 1$, $v(1) := \pi \sim unif[0.4, 1]$, and $v(2) = 0$. Fix $\delta = 0.8$, and suppose that Player 2 is the first proposer. Suppose that:

- Player 2 proposes N, and Player 1 always makes a non-informative proposal that is rejected by Player 2.
- Player 2 accepts a proposal to form N if and only if it offers her an amount at least as great as δ times the maximum expected payoff that she can obtain by making a proposal herself. Any type 1_π accepts a proposal if it offers her at least π, or else she quits.

We now show informally that when all players are expected to play in a manner consistent with the description above, no player can benefit by deviating unilaterally. To see this, note that proposing $N = \{1, 2\}$ is better than quitting for Player 2, if the former

[25] [29] considers an incomplete information setting where types eventually become verifiable, which is not the case in our paper.

gives a positive expected payoff. Indeed, the expected payoff to Player 2 from proposing N and offering y_1 amount to the informed party 1 is $(1 - y_1)Prob(\pi \leq y_1) = \frac{(1-y_1)(y_1-0.4)}{0.6}$ which is maximized when $y_1 = 0.7$ giving Player 2 a positive equilibrium expected payoff of $\frac{0.09}{0.6} = 0.15$. Now, consider the informed party 1. If Player 1's proposal is informative on the equilibrium path, then the belief support at the subsequent continuation game would get divided into at least two subintervals, leading to profitability of informed types mimicking each other, which would contradict equilibrium. We explain this argument in greater detail in the following example.

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